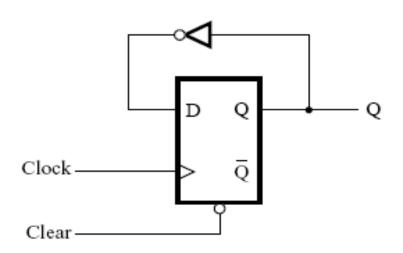
BEE 271 Digital circuits and systems Spring 2017 Lecture 12: State machines

Nicole Hamilton https://faculty.washington.edu/kd1uj

Today's topics

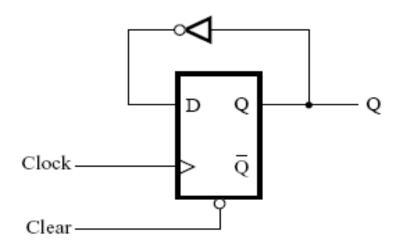
- 1. Review
- 2. State machines



Assume: t_{su} = 0.6 ns $t_{\rm h} = 0.4 \, \rm ns$ 0.8 ns <= t_{cQ} <= 1.0 ns $t_{gate} = 1.0 + 0.1k$ where k = number of inputs $T_{min} = t_{cOmax} + t_{NOT} + t_{su}$ = 1.0 + 1.1 + 0.6 = 2.7 ns

F_{max} = 1/2.7 ns = 370.37 MHz.

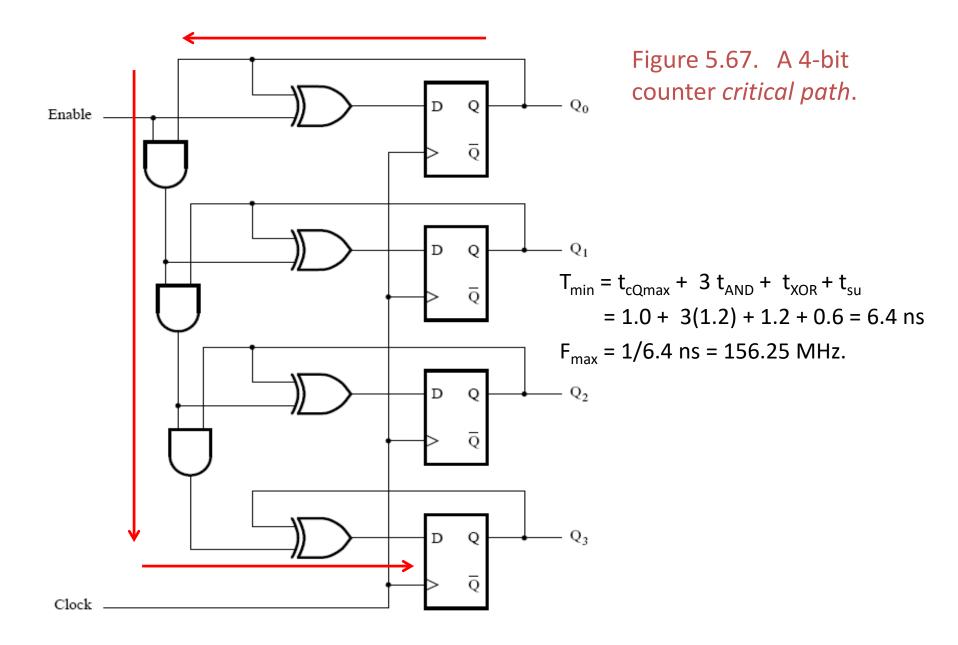
Figure 5.66. A simple flip-flop circuit.

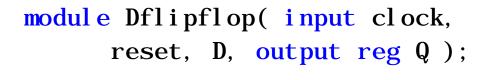


Checking hold time Assume: $t_{su} = 0.6 \text{ ns}$ $t_{h} = 0.4 \text{ ns}$ $0.8 \text{ ns} <= t_{cQ} <= 1.0 \text{ ns}$ $t_{gate} = 1.0 + 0.1 \text{k}$ where k = number of inputs

Shortest delay = $t_{cQmin} + t_{NOT}$ = 0.8 + 1.1 = 1.9 ns Since 1.9 ns > t_h = 0.4 ns, no hold violation.

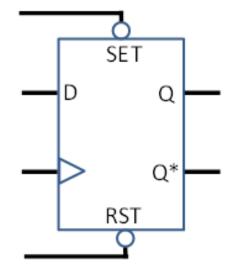
Figure 5.66. A simple flip-flop circuit.





always @(posedge clock)
 Q <= reset ? 0 : D;</pre>

endmodul e



Edge-triggered

Code for a D flip-flop using a <= non-blocking assignment.

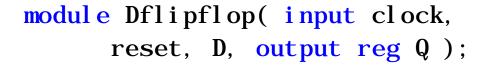
module Dflipflop(input clock, reset, D, output reg Q);

always @(posedge clock)
 Q <= reset ? 0 : D;</pre>

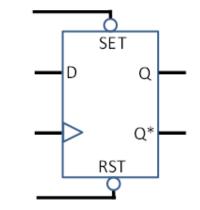
endmodul e

The RHS of the <= operator is evaluated just before the clock edge and the assignment is made just after the clock edge.

Code for a D flip-flop using a <= non-blocking assignment.

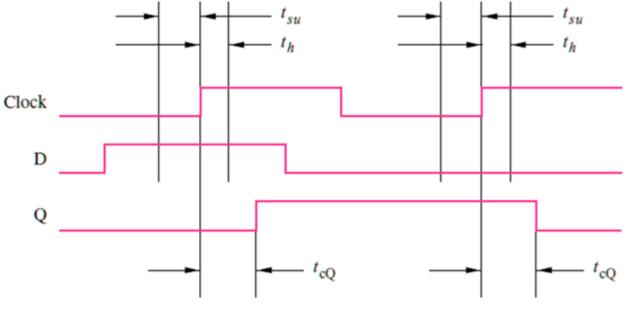


```
always @( posedge clock )
  Q <= reset ? 0 : D;</pre>
```



endmodul e

Edge-triggered

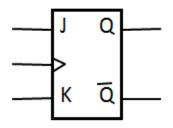


(b) Timing diagram

```
module JK( input clock, J, K, reset,
      output reg Q );
   always @( posedge reset,
         posedge clock )
      casex ( { reset, J, K } )
         'b1xx: Q <= 0;
         'b000: Q <= Q;
         'b001: Q <= 0;
         'b010: Q <= 1;
         'b011: Q <= ~Q;
      endcase
```

 $\begin{array}{c|c|c} J & K & Q (t+1) \\ \hline 0 & 0 & Q (t) \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & \overline{Q} (t) \end{array}$

(b) Truth table



endmodule

(c) Graphical symbol

```
module SynchReset( input clock, reset, D,
        output Q );
     // Synchronous reset (synchronized to the clock)
     always @( posedge clock )
        Q <= reset ? 0 : D;</pre>
```

endmodule

module AsyncReset(input clock, reset, D, output Q);

// Asynchronous reset (not synchronized to the clock)
always @(posedge reset, posedge clock)
 Q <= reset ? 0 : D;</pre>

endmodule

D flip-flops with synchronous and asynchronous resets.

```
module CounterA(
    input clock, reset,
    input [ 31:0 ] resetValue,
    output reg [ 31:0 ] count = 0 );
// Synchronous reset (synchronized to the clock)
    always @( posedge clock )
    count <= reset ? resetValue : count + 1;</pre>
```

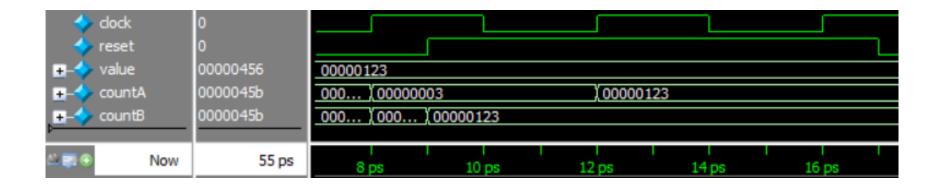
endmodule

```
module CounterB(
    input clock, reset,
    input [ 31:0 ] resetValue,
    output reg [ 31:0 ] count = 0 );
```

// Asynchronous reset (not synchronized to the clock)
always @(posedge reset, posedge clock)
 count <= reset ? resetValue : count + 1;</pre>

endmodule

From simulation, reset in *synchronous* in CounterA, changing only *with* the clock, and *asynchronous* in CounterB.



CounterA

```
always @( posedge clock )
    count <= reset ?
    resetValue : count + 1;</pre>
```

CounterB

```
always @( posedge reset, posedge clock )
    count <= reset ?
        resetValue : count + 1;</pre>
```

```
module ShiftRegister #( parameter n = 4 )
      ( input clock, reset, D,
      output reg [0 : n - 1] Q );
   always @( posedge clock )
      if ( reset )
         Q <= 0;
      else
         begin
         integer i;
         for (i = n - 1; i! = 0; i = i - 1)
            Q[ i ] <= Q[ i - 1 ];
         Q[ 0 ] <= D;
         end
```

endmodule

```
module ShiftRegister2 #( parameter n = 4 )
        ( input clock, reset, D,
        output reg [ 0 : n - 1 ] Q );
        always @( posedge clock )
        if ( reset )
            Q <= 0;
        else
            Q <= { D, Q[ 0 : n - 2 ] };
endmodule</pre>
```

Finite state machines

Machines whose next state depends on the inputs and the previous state.

Called finite state machines because they have only a *finite* number of states.

In a combinational circuit, the values of the outputs are determined solely by the present values of its inputs.

In a sequential circuit, the values of the outputs depend on the past behavior of the circuit, as well as the present values of its inputs.

A sequential circuit has states, which in conjunction with the present values of inputs determine its behavior. Sequential circuits can be:

• Synchronous – where flip-flops are used to implement the states, and a clock signal is used to control the operation

• Asynchronous – where no clock is used

Terminology

Sequential circuits: Outputs depend on the past behavior as well as the present inputs.

Synchronous sequential: Controlled by a clock.

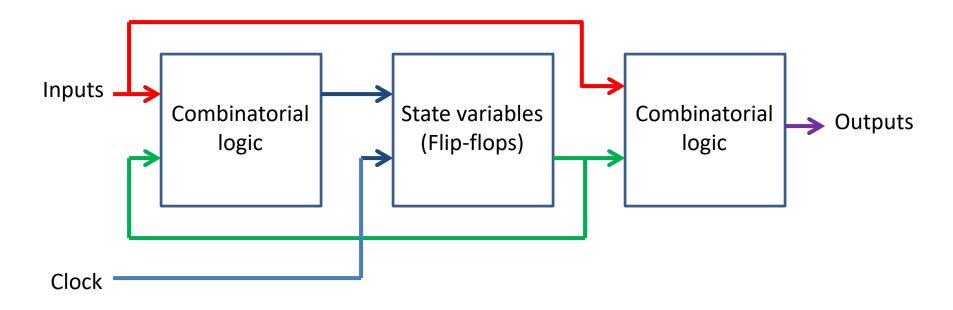
Asynchronous sequential: No clock is used.

State: The stored values of any flip-flops.

Active edge: The edge of the clock that causes the outputs to change.

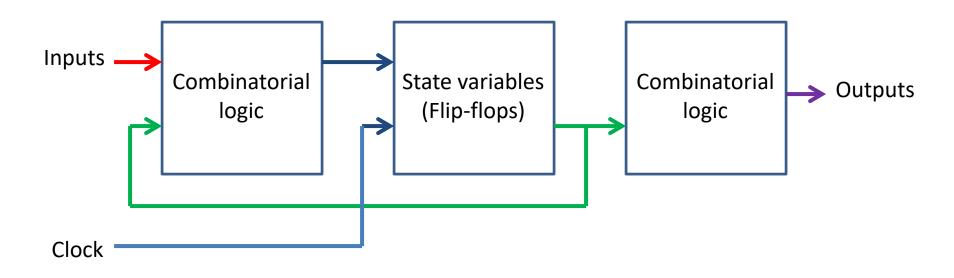
Finite state machine (FSM): Formal name for a sequential circuit.

Generalized form of a sequential circuit

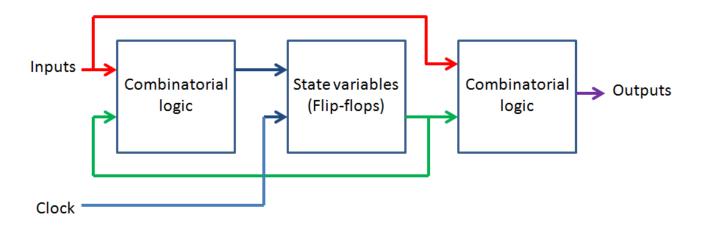


If the outputs depend on both the current state and the current inputs, it is called a *Mealy* machine, named after George Mealy, who invented the concept in 1955.

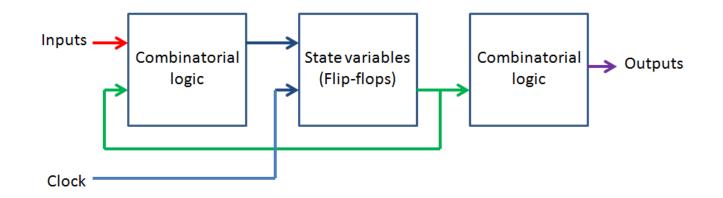
Moore machine



If the outputs depend only on the current state, it is called a **Moore** machine. ("Moore is less.") It's named after Edward Moore, who invented the concept in 1956.



Mealy machines require fewer states but if the inputs change asynchronously, the outputs can change asynchronously as well.



Moore machines require more state variables and the outputs are delayed by one clock. But all the outputs are guaranteed to be synchronous.

Asynchronous inputs and metastability

Two related problems.

- 1. *Asynchronous inputs.* An input may change be between clocks.
- Metastability. The input might not actually be a 1 or a 0.

If an input signal with one of these problems is a combinatorial factor in your output, your output will have the same problems.

Solution

We can condition the input somewhat with analog circuitry, e.g., Schmidt triggers.

The digital solution is that we can sample it with a clock and save the result in a flip-flop.

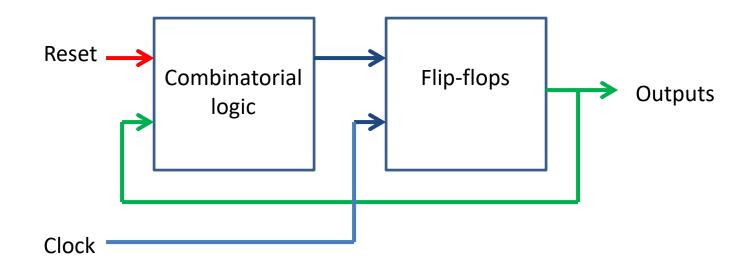
We expect that no matter what the input, the flip-flop will certainly settle to 1 or 0 and that's usually true.

But note

Sampling doesn't always work.

- It's possible for a flip-flop to get stuck in a metastable state because of a bad input, never settling into a 0 or 1.
 - If you add additional flip-flops to create a shift register, you can reduce the likelihood of having a metastable circuit but you cannot make it go away.

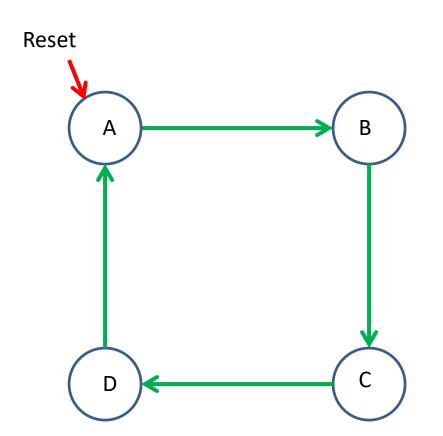
Simplest example: A counter



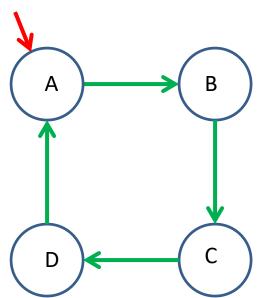
State diagram for a counter with 4 states.

At each clock, it always moves to the next state.

The arrows between states are called *edges* or *transitions*.







State table

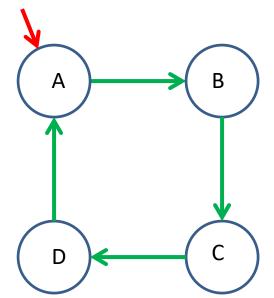
Present state	Next state
А	В
В	С
С	D
D	A

We pick any state assignments we like, though some could be better than others.

State-assigned table

	Present state	Next state
	y2 y1	Y2 Y1
А	00	01
В	01	10
С	10	11
D	11	00





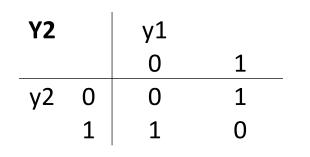
State-assigned table

	Present state	Next state
	y2 y1	Y2 Y1
А	00	01
В	01	10
С	10	11
D	11	00

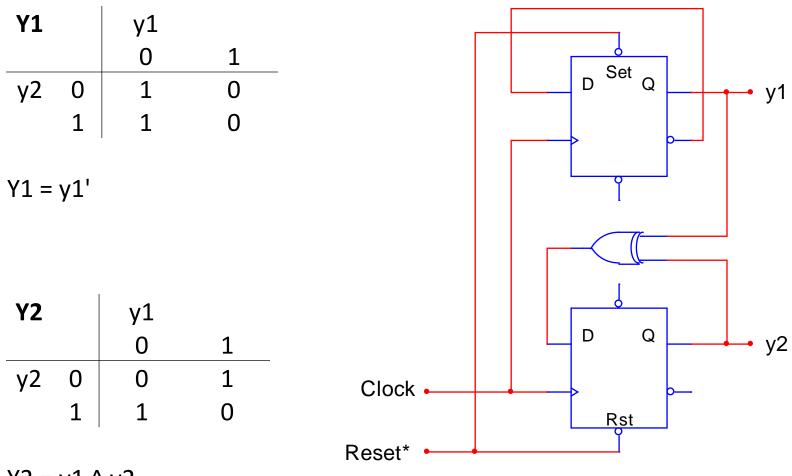
Having picked the assignments, we can use Karnaugh maps to derive the equations for the next state variables.

Y1		y1	
		0	1
y2	0	1	0
	1	1	0

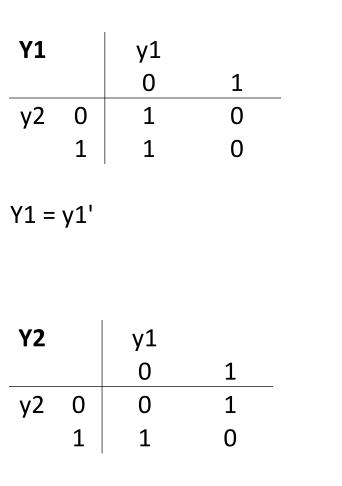
Y1 = y1'



Y2 = y1 ^ y2



Y2 = y1 ^ y2



 $Y2 = y1^{y2}$

module Counter(input clock, reset, output reg y1, y2); always @(posedge reset, posedge clock) if (reset) begin y1 <= 0; y2 <= **0**; end else begin y1 <= ~y1; y2 <= y1 ^ y2; end endmodule

In Verilog, the <= "non-blocking" assignment means all the assignments happen synchronously at exit from the always block. In Verilog, we would probably skip the Karnaugh maps write the code directly from the state-assigned table.

State-assigned table

	Present state	Next state
	y2 y1	Y2 Y1
А	00	01
В	01	10
С	10	11
D	11	00

```
module Counter2( input clock,
    reset,
    output reg [ 1:0 ] y );
    always @( posedge reset,
        posedge clock )
    y <= reset ? 0 : y + 1;
endmodule
```

If the cases were more complex or not in order, we might write it like this with a case statement.

State-assigned table

	Present state	Next state
	y2 y1	Y2 Y1
А	00	01
В	01	10
С	10	11
D	11	00

```
module Counter3( input clock,
    reset,
    output reg [ 1:0 ] y );
```

```
always @( posedge reset,
    posedge clock )
    if ( reset )
        y <= 0;
    else
        case ( y )
        0: y <= 1;
        1: y <= 2;
        2: y <= 3;
        3: y <= 0;
        endcase
```

endmodule

Or we might parameterize the assignments.

State-assigned table	

Present state	Next state
y2 y1	Y2 Y1
00	01
01	10
10	11
11	00
	y2 y1 00 01 10

```
module Counter4( input clock,
    reset,
    output reg [ 1:0 ] y );
parameter A = 0, B = 1,
    C = 2, D = 3;
```

endmodule

Verilog makes it really easy to pick any assignments you like and walk from one arbitrary state to another.

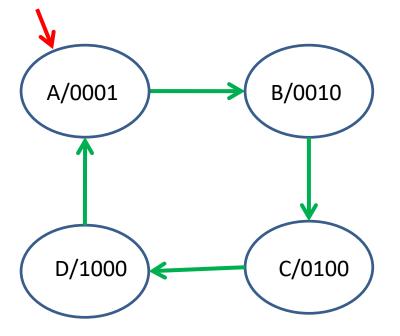
Example: Create a 3-bit counter in Verilog that cycles through this sequence: 4, 7, 0, 3, 2, 6, 1, 5.

```
Example: Create a 3-bit counter in Verilog that cycles through this sequence: 4, 7, 0, 3, 2, 6, 1, 5.
```

```
module PseudoRandom( input clock,
    output reg [ 2:0 ] Q );
  always @( posedge clock )
    case (Q)
      4: Q <= 7;
      7: Q \ll 0;
      0: Q <= 3;
      3: Q <= 2;
      2: Q <= 6;
      6: Q <= 1;
      1: Q \ll 5;
      5: Q <= 4;
    endcase
```

endmodule

Reset

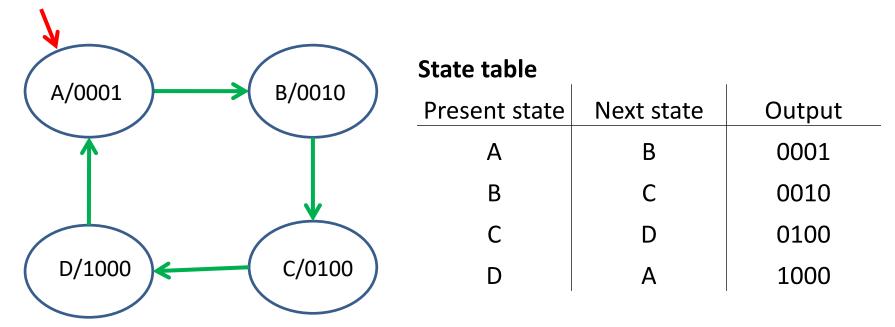


In the previous example, we didn't specify the outputs, merely that it had to count.

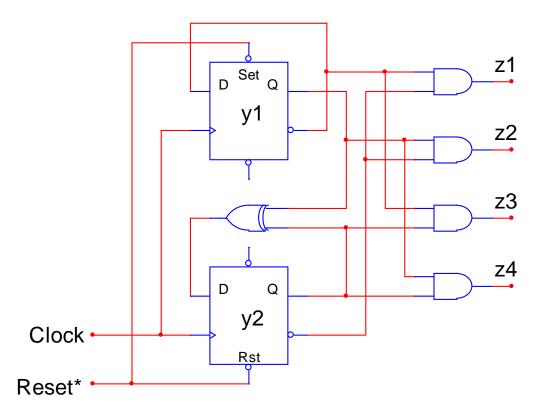
But consider this state diagram, where the outputs have been specified.

The "/nnnn" part specifies the desired outputs at each state.

Since the outputs depend only on the state, this is a Moore machine. Reset

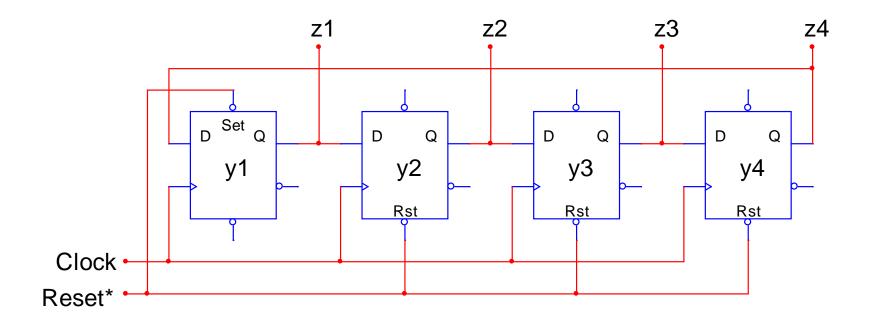


State-assigned table										
		Present state	Next state	Output						
Suppose we use		y2 y1	Y2 Y1	z4 z3 z2 z1						
the previous state	А	00	01	0001						
assignments.	В	01	10	0010						
	С	10	11	0100						
	D	11	00	1000						



State-assigned table

		Present state	Next state	Output
With these		y2 y1	Y2 Y1	z4 z3 z2 z1
assignments, we	А	00	01	0001
need a decoder	В	01	10	0010
to produce the	С	10	11	0100
outputs.	D	11	00	1000

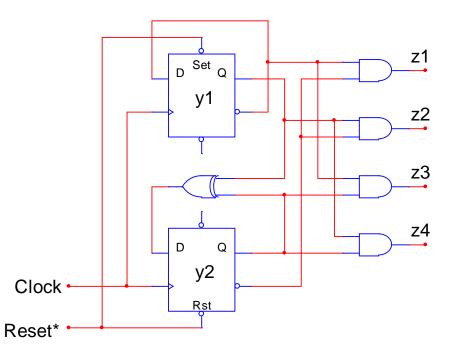


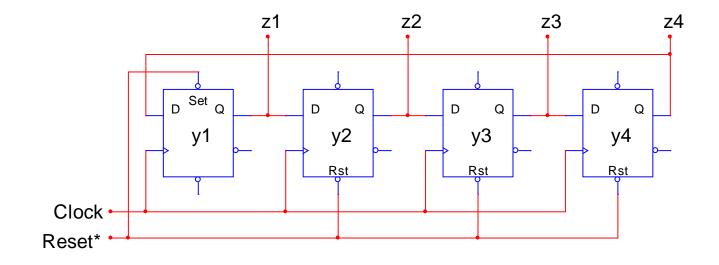
"One hot" state-assigned table

		Present state	Next state	Output
Here is an alternate		y4 y3 y2 y1	Y4 Y3 Y2 Y1	z4 z3 z2 z1
assignment as a	А	0001	0010	0001
"one-hot" ring	В	0010	0100	0010
counter.	С	0100	1000	0100
	D	1000	0001	1000

So which is the better design? Both are correct.

Picking a good set of state assignments usually involves consideration of both cost and elegance.





Example: A simple speed control

Outputs 1 if a vehicle's speed is excessive for 2 or more clocks.

Example: A speed governor that limits a vehicle's top speed.

Input:

w == 1 \rightarrow excessive speed

w == 0 \rightarrow speed acceptable

Output:

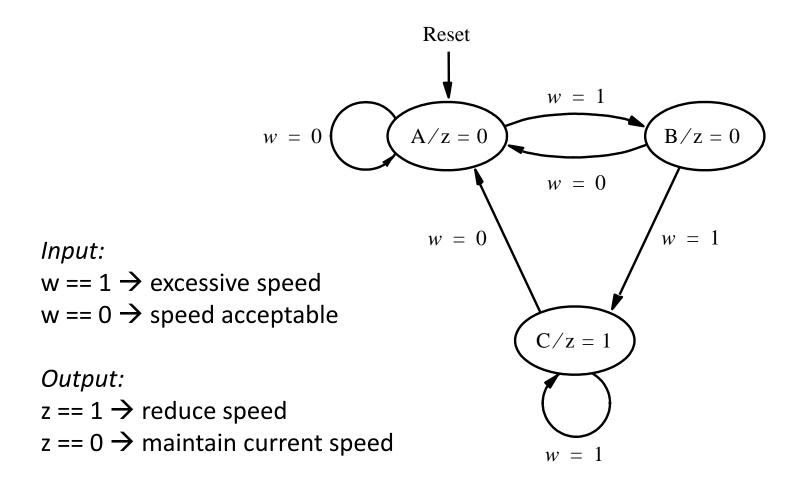
 $z == 1 \rightarrow$ reduce speed

 $z == 0 \rightarrow$ maintain current speed

w == 1 for 2+ clocks \rightarrow z == 1

Clockcycle: w: z:	t ₀	t ₁	t ₂	t ₃	t ₄	t ₅	t ₆	t ₇	t ₈	t9	t ₁₀
<i>w</i> :	0	1	0	1	1	0	1	1	1	0	1
Z:	0	0	0	0	0	1	0	0	1	1	0

Figure 6.2. Sequences of input and output signals.



w == 1 for 2+ clocks \rightarrow z == 1

Figure 6.3. State diagram for the speed control.

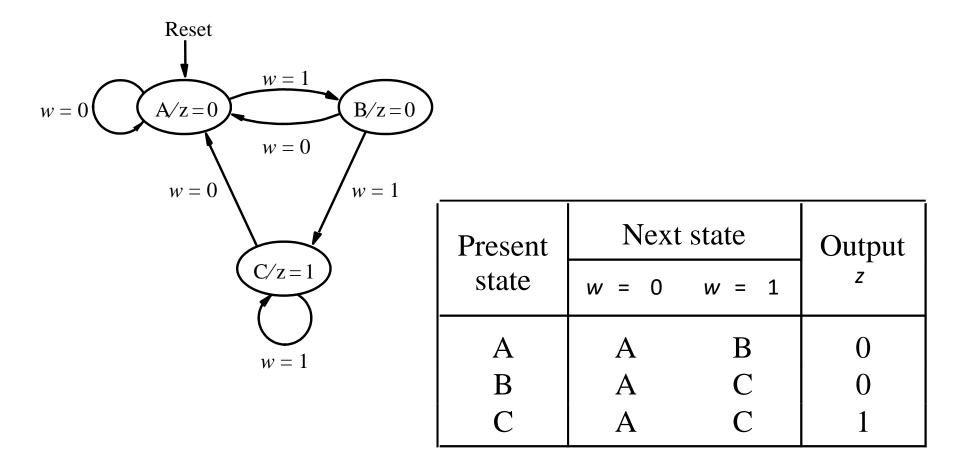


Figure 6.4. State table for the speed control.

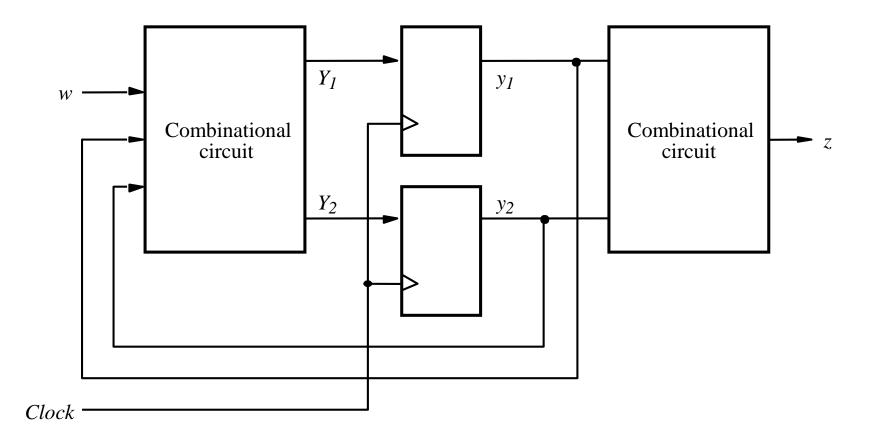
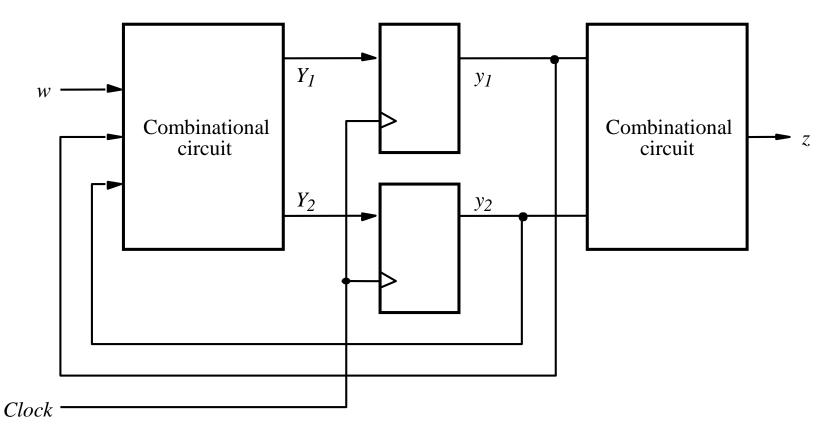


Figure 6.5. A generalized solution to the speed controller, with input *w*, output *z*, and two flip-flops for the three states.



The present state variables, y_1 and y_2 , determine the present state of the circuit.

The next state variables, Y_1 and Y_2 , determine the state into which the circuit will go after the next active edge of the clock signal.

State variable assignments

Each of the states in a state diagram or a state table must be represented by some unique combination of 1's and 0's.

We have to pick those assignments and some assignments are better than others.

w = 0 $A/z = 0$ $W = 1$ B/z	z=0)			
w = 0 $w =$	= 1	Present	Next s	tate	
C/z=1		state	<i>w</i> = 0	w = 1	Output
\bigcirc		^y 2 ^y 1	^Y 2 ^Y 1	^Y 2 ^Y 1	Z
w = 1	A	00	00	01	0
	В	01	00	10	0
	С	10	00	10	1
		11	dd	dd	d

Figure 6.6. One possible state assignment for the speed controller.

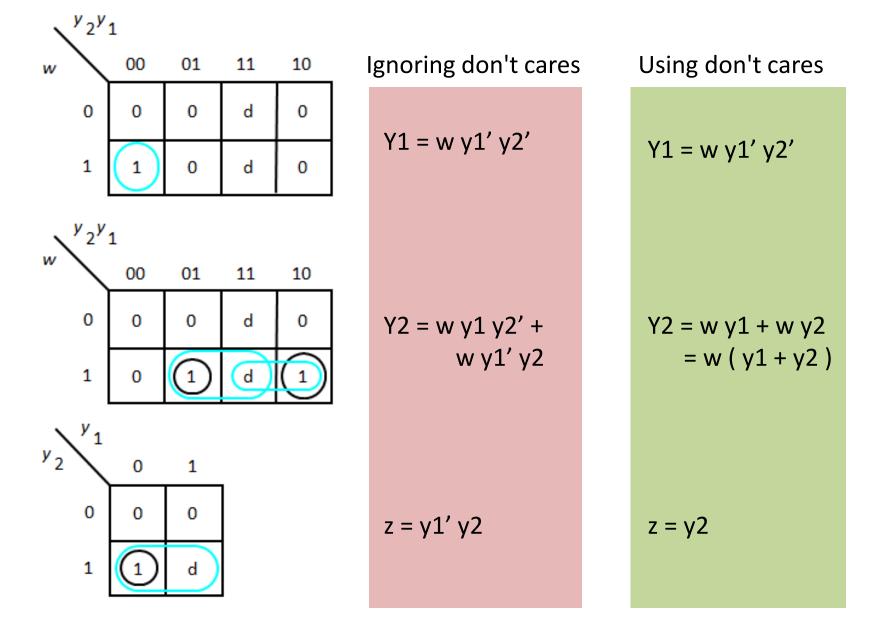


Figure 6.7. Karnaugh maps for the next state variables in the speed controller.

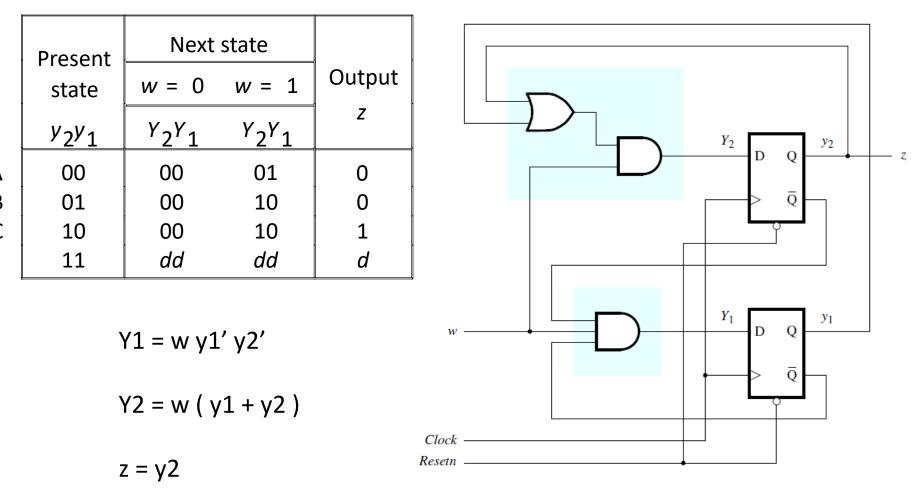


Figure 6.8. Final implementation of the speed controller using the don't cares.

A B C

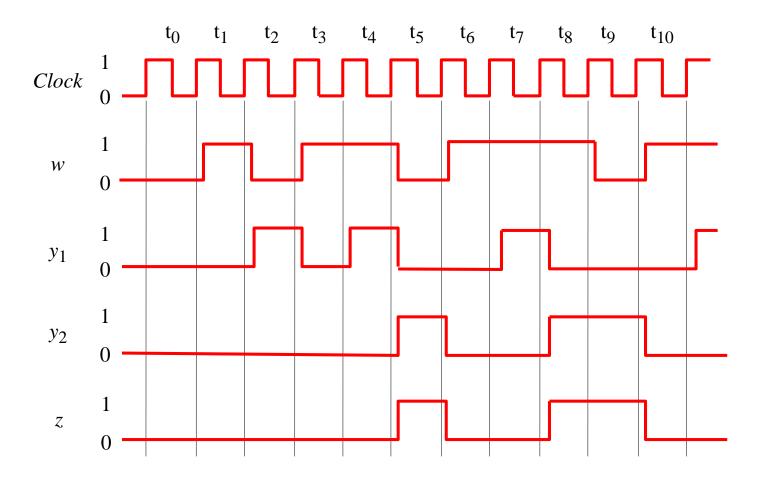


Figure 6.9. Timing diagram for the speed controller.

Design steps:

- 1. Obtain the specification of the desired circuit.
- 2. Derive a state diagram.
- 3. Derive the corresponding state table.
- 4. Reduce the number of states if possible.
- 5. Decide on the number of state variables.
- 6. Choose the type of flip-flops to be used.
- 7. Derive the logic expressions needed to implement the circuit.

Are all state assignments equivalent?

No, they are not.

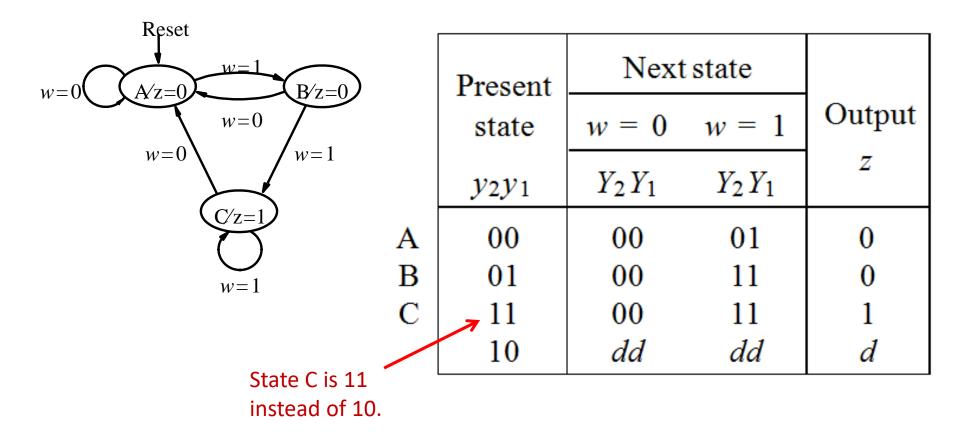


Figure 6.16. Improved state assignment for the speed controller.

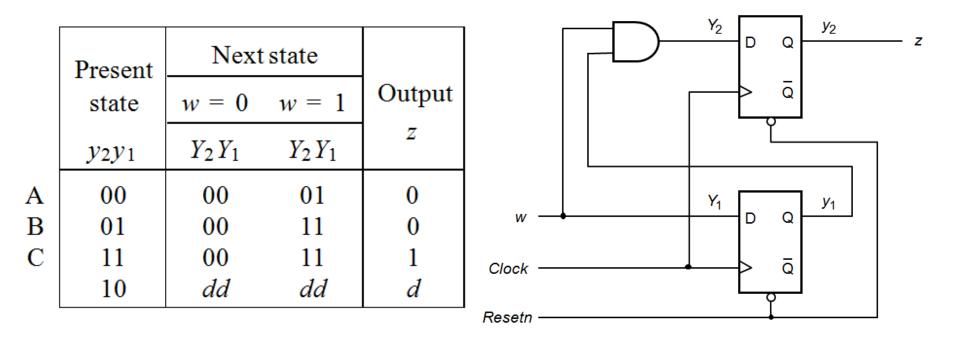
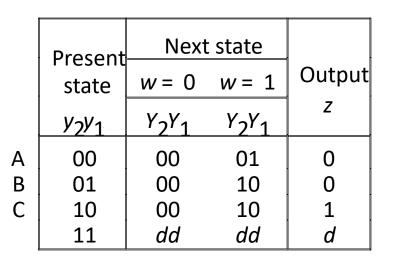
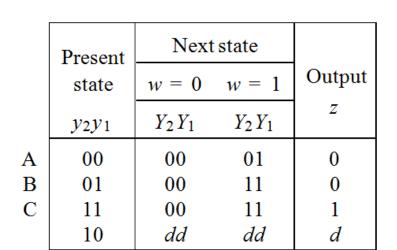
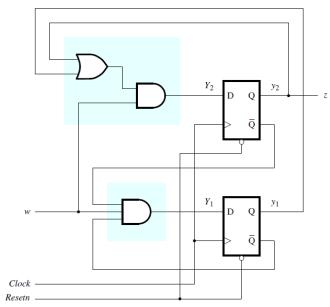
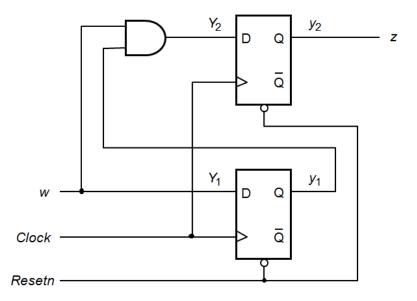


Figure 6.17. Final circuit for the improved state assignment for the speed controller.

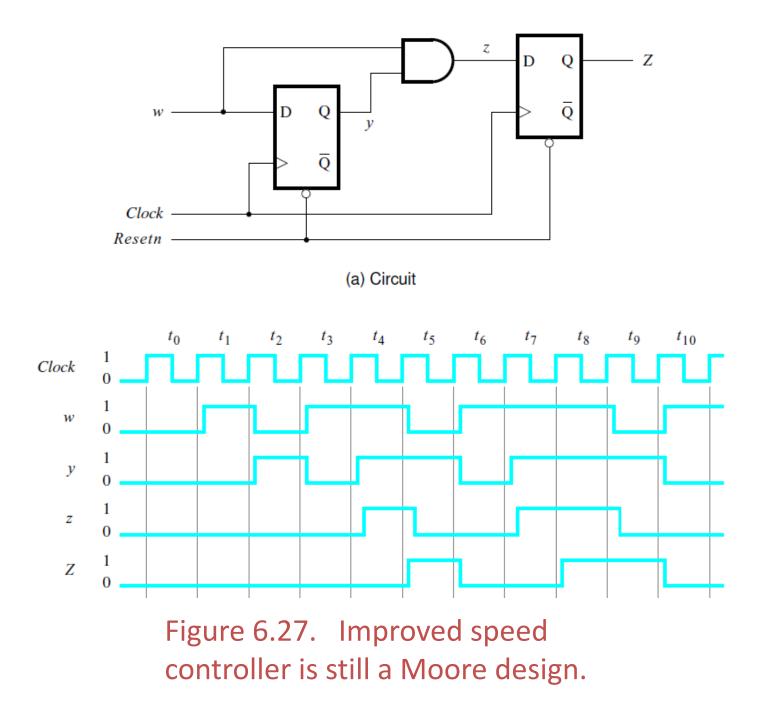








Original versus improved state assignment for the speed controller.



```
module simple1 ( input clock,
    reset, w, output z );
reg [ 2:1 ] y, Y;
parameter [ 2:1 ] A = 2'b00,
    B = 2'b01, C = 2'b10;
```

```
// Define the next state
always @( w, y )
    case ( y )
    A: if ( w ) Y = B;
    else Y = A;
    B: if ( w ) Y = C;
    else Y = A;
    C: if ( w ) Y = C;
    else Y = A;
    default: Y = 2'bxx;
endcase
```

// Define the sequential block
always @(posedge reset,
 posedge clock)
 if (reset) y <= A;
 else y <= Y;</pre>

// Define output
assign z = y == C;

endmodule

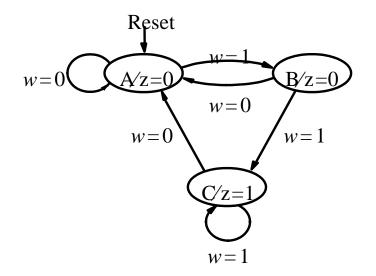
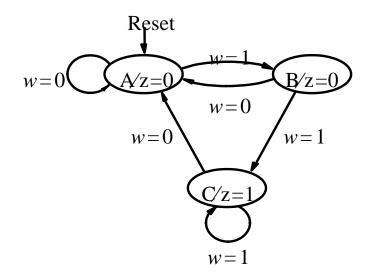


Figure 6.29. Verilog code for the speed controller.



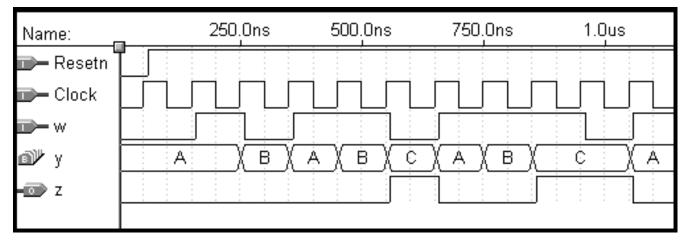


Figure 6.32. Simulation results for the speed controller.

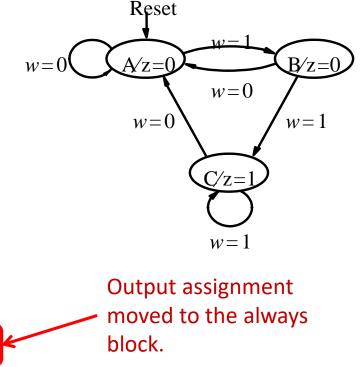
```
reset, w, output reg z );
```

```
reg [ 2:1 ] y, Y;
parameter [2:1] A = 2'b00,
   B = 2'b01, C = 2'b10;
```

```
// Define the next state
always @(w, y)
   begin
   case (y)
     A: if (w) Y = B;
        else Y = A;
     B: if (w) Y = C;
        else Y = A;
     C: if (w) Y = C;
        else Y = A;
     default: Y = 2'bxx;
   endcase
   z = y == C; // Define output
   end
```

```
module simple2 ( input clock, // Define the sequential block
                                         always @( posedge reset,
                                               posedge clock)
                                            if ( reset ) y <= A;</pre>
                                            else y <= Y;</pre>
```

endmodule

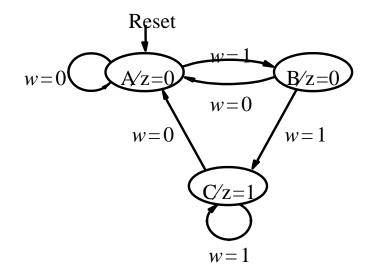


```
assign z = y == C;
      reset, w, output z );
                                  endmodule
   reg [ 2:1 ] y, Y;
   parameter [2:1] A = 2'b00,
      B = 2'b01, C = 2'b10;
                                       Next state and reset
                                       calculations moved into
   // Define the next state
                                       the always block.
   always @( posedge reset,
        posedge clock)
                                           Reset
      if (reset)
        y <= A;
                                                       B/z=
                                   w \equiv 0
     else
                                                 w = 0
        case (y)
                                                       w = 1
                                           w=0
           A: if ( w ) y <= B;
              else y <= A;
                                                 ~/z=
           B: if ( w ) y <= C;
              else y <= A;
                                                 w = 1
           C: if ( w ) y <= C;
              else y <= A;
           default: y <= 2'bxx;</pre>
        endcase
                                             Figure 6.34
```

```
module simple4 ( input clock,
      reset, w, output reg z );
   reg [ 2:1 ] y, Y;
   parameter [2:1] A = 2'b00,
      B = 2'b01, C = 2'b10;
   // Define the next state
   always @( posedge reset,
         posedge clock)
      casex ( { reset, w, y } )
         3'b1xxx: y <= A;
         3'b00xx: y <= A;
         { 2'b01, A }: y <= B;
         { 2'b01, B }: y <= C;
         { 2'b01, C }: y <= C;
         default: y <= 2'bxx;</pre>
      endcase
```

// Define output
 assign z = y == C;

endmodule



```
module simple5 ( input clock,
    reset, w, output reg z );
reg [ 2:1 ] y, Y;
```

```
parameter [ 2:1 ] A = 2'b00,

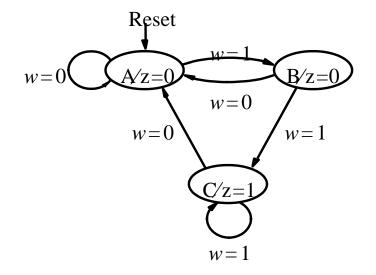
B = 2'b01, C = 2'b10,

x = 2'bxx;
```

// Define the next state
always @(posedge reset,
 posedge clock)
 casex ({ reset, w, y })
 { 2'b1x, x }: y <= A;
 { 2'b00, x }: y <= A;
 { 2'b01, A }: y <= B;
 { 2'b01, B }: y <= C;
 { 2'b01, C }: y <= C;
 default: y <= x;</pre>

// Define output
 assign z = y == C;

endmodule



endcase

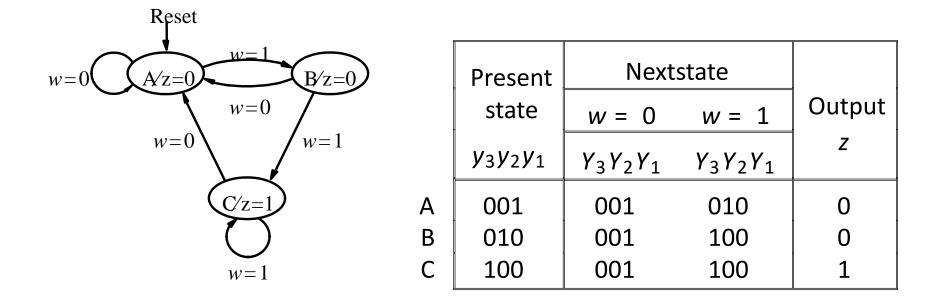


Figure 6.20. Anther alternative: A one-hot state assignment for the speed controller.

A Mealy alternative

Clockcycle:	t ₀	t_1	t ₂	t ₃	t4	t ₅	t ₆	t ₇	t ₈	t9	t ₁₀
w:											1
<i>z</i> :	0	0	0	0	0	1	0	0	1	1	0

The original Moore design

Clock cycle: w: z:	t ₀	t_1	t ₂	t ₃	t4	t ₅	t ₆	t ₇	t ₈	t9	t ₁₀
w:	0	1	0	1	1	0	1	1	1	0	1
<i>z</i> :	0	0	0	0	1	0	0	1	1	0	0

The alternative Mealy design

Figure 6.22. Sequence detector for an alternate speed controller. This one recognizes w = 1 on two cycles immediately.

Clock cycle: w: z:	t ₀	t ₁	t ₂	t ₃	t4	t5	t ₆	t ₇	t ₈	t9	t ₁₀
w:	0	1	0	1	1	0	1	1	1	0	1
<i>z</i> :	0	0	0	0	1	0	0	1	1	0	0

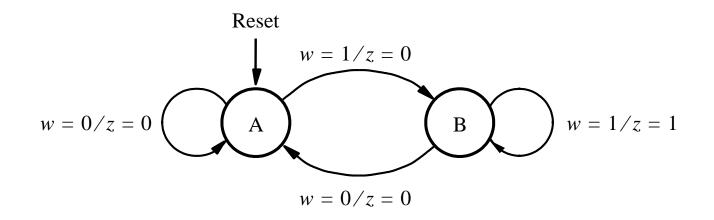
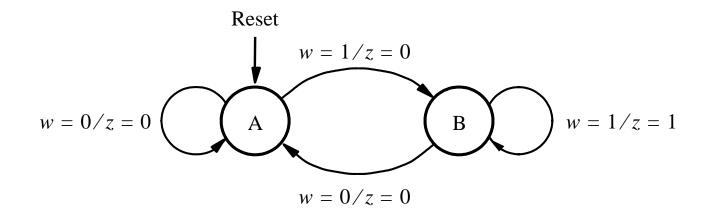


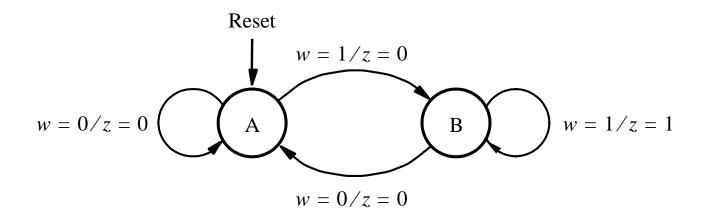
Figure 6.23. State diagram of the alternate speed controller.

Clock cycle: <i>w</i> :	t ₀	t_1	t ₂	t ₃	t4	t ₅	t ₆	t ₇	t ₈	t9	t ₁₀
w:	0	1	0	1	1	0	1	1	1	0	1
<i>z</i> :	0	0	0	0	1	0	0	1	1	0	0



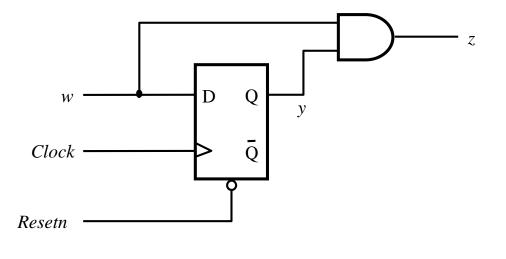
Present	Next	state	Output z		
state	<i>w</i> = 0	<i>w</i> = 1	<i>w</i> = 0	w = 1	
А	А	В	0	0	
В	А	В	0	1	

Figure 6.24. State table for the alternate speed controller.



	Present	Next	state	Output			
	state	<i>w</i> = 0	<i>w</i> = 1	<i>w</i> = 0	<i>w</i> = 1		
	У	Y	Y Y		Z		
А	0	0	1	0	0		
В	1	0	1	0	1		

Figure 6.25. State-assigned table for an alternate Mealy speed controller.



(a) Circuit

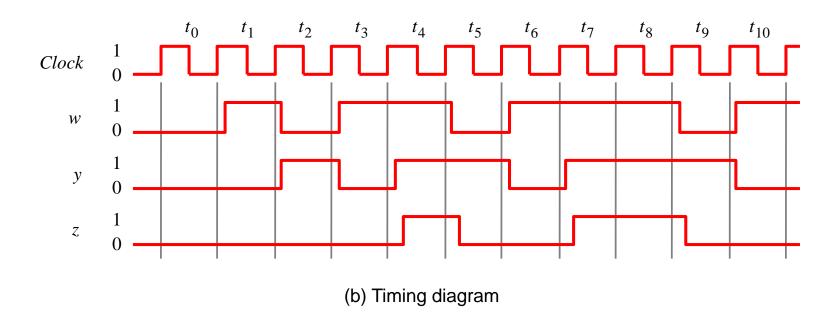
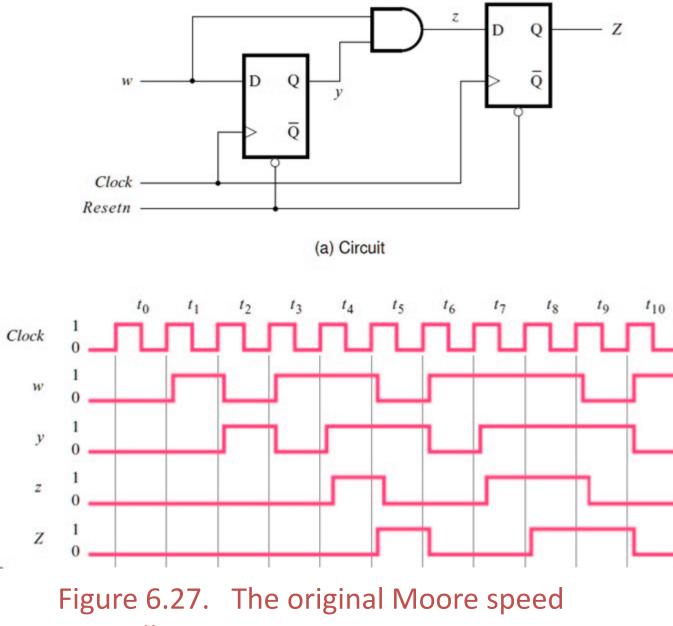
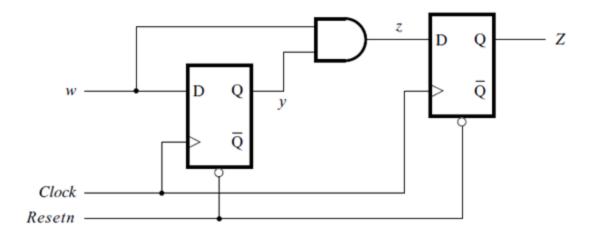


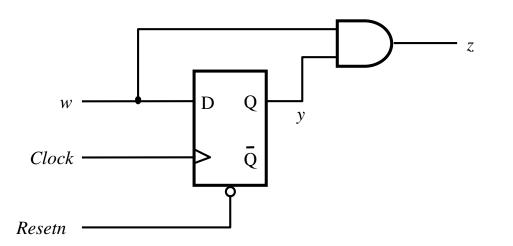
Figure 6.26. The alternate Mealy speed controller.



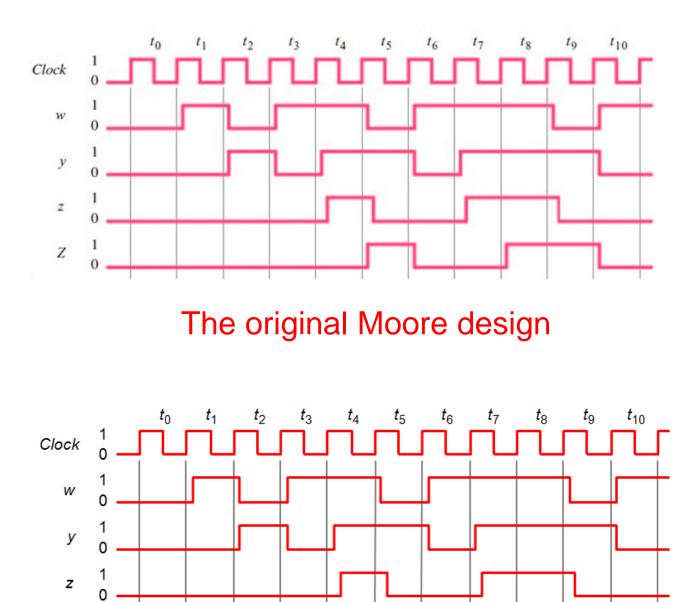
controller.



The original Moore design



The alternative Mealy design



The alternative Mealy design

```
module mealy1( input clock, reset, w,
                                                        B: if (w)
                                                              begin
      output reg z );
                                                              z = 1;
                                                              Y = B;
   reg y, Y;
   parameter A = 0, B = 1;
                                                              end
                                                           else
   // Define the next state and outputs
                                                              begin
   always @( * )
                                                              z = 0;
      case ( y )
                                                              Y = A;
         A: if (w)
                                                              end
                                                        endcase
                begin
                z = 0;
                Y = B;
                                                 // Define the sequential block
                end
                                                 always @( posedge reset,
                                                        posedge clock )
            else
                                                     if (reset)
                begin
                z = 0;
                                                        y <= A;
                Y = A;
                                                     else
                end
                                                        y <= Y;
                       Reset
                                              endmodule
                            w = 1/z = 0
                                             w = 1/z = 1
           w = 0/z = 0
                                      в
                                                            Figure 6.36
                            w = 0/z = 0
```

```
module mealy2( input clock, reset, w,
                                               // Define the sequential block
                                                 always @( posedge reset,
      output reg z );
                                                        posedge clock )
   reg y, Y;
                                                    if (reset)
   parameter A = 0, B = 1;
                                                        y <= A;
                                                    else
   // Define the next state and outputs
                                                        y <= Y;
   always @( * )
      if ( ~w )
                                              endmodule
         begin
         z = 0;
         Y = A;
         end
                                          Reset
      else
                                                w = 1/z = 0
         case (y)
            A: begin
                          w = 0/z = 0
                                                                     w = 1/z = 1
                                           А
                                                            В
               z = 0;
               Y = B;
               end
                                                w = 0/z = 0
            B: begin
               z = 1;
               Y = B;
               end
         endcase
```

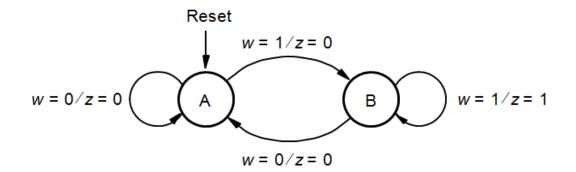
```
module mealy3( input clock, reset, w,
      output z );
   reg y, Y;
   parameter A = 0, B = 1;
   // Define the next state
   always @( * )
      if ( ~w )
                                             Reset
         Y = A;
      else
                                                  w = 1/z = 0
         case ( y )
            A: Y = B;
                              w=0/z=0
                                                                       w = 1/z = 1
                                              Α
                                                              В
            B: Y = B;
         endcase
                                                  w = 0/z = 0
   assign z = y == B \& w;
   // Define the sequential block
   always @( posedge reset,
         posedge clock )
      y \ll reset ? A : Y;
```

endmodule

```
module mealy4( input clock, reset, w,
        output z );
reg y;
parameter A = 0, B = 1;
always @( posedge reset,
        posedge clock )
        y <= ( reset | ~w ) ? A : B;</pre>
```

assign z = y == B & w;

endmodule



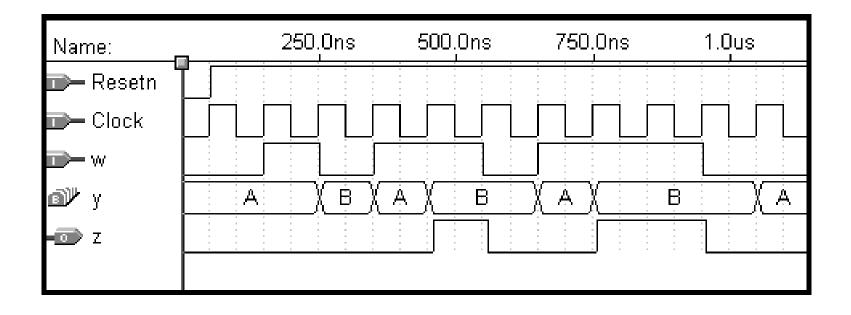


Figure 6.37. Simulation results for the Mealy machine.

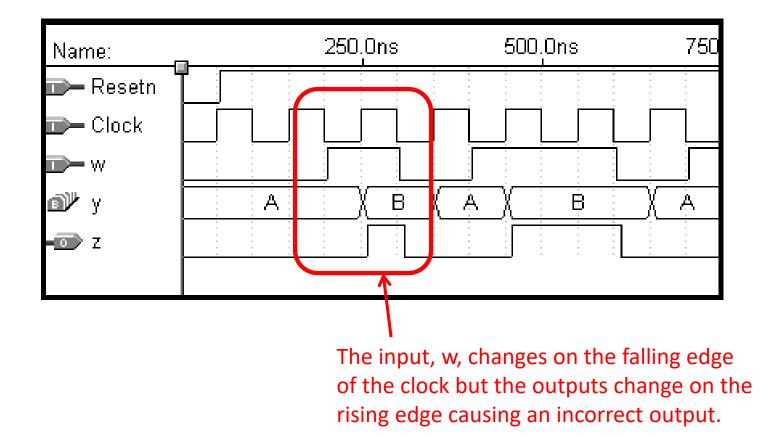


Figure 6.38. Potential problem with asynchronous inputs to the Mealy speed controller.