### BEE 271 Digital circuits and systems Spring 2017 Lecture 12: State machines

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# Today's topics

- 1. Review
- 2. State machines



Assume:  $t_{\rm su} = 0.6$  ns  $t_h = 0.4$  ns 0.8 ns  $\lt = t_{cQ} \lt = 1.0$  ns  $t_{gate} = 1.0 + 0.1k$ where  $k =$  number of inputs  $T_{min} = t_{cOmax} + t_{NOT} + t_{su}$  $= 1.0 + 1.1 + 0.6 = 2.7$  ns  $F_{\text{max}} = 1/2.7$  ns = 370.37 MHz.

Figure 5.66. A simple flip-flop circuit.



Checking hold time Assume:  $t_{\rm su} = 0.6$  ns  $t_h = 0.4$  ns 0.8 ns  $\lt = t_{cQ} \lt = 1.0$  ns  $t_{gate} = 1.0 + 0.1k$ where  $k =$  number of inputs

Shortest delay =  $t_{\text{cQmin}} + t_{\text{NOT}}$  $= 0.8 + 1.1 = 1.9$  ns Since 1.9 ns  $> t_h = 0.4$  ns, no hold violation.

Figure 5.66. A simple flip-flop circuit.





always @( posedge clock )  $Q \le$  reset ?  $0 : D$ ;

endmodule



Edge-triggered

Code for a D flip-flop using a <= non-blocking assignment.

module Dflipflop( input clock, reset,  $D$ , output reg  $Q$ );

always @( posedge clock )  $Q \le$  reset ?  $0 : D$ ;

endmodule

The RHS of the <= operator is evaluated just before the clock edge and the assignment is made just after the clock edge.

Code for a D flip-flop using a <= non-blocking assignment.



```
always @( posedge clock )
   Q \leq \text{reset} ? 0 : D;
```


#### endmodule

Edge-triggered



(b) Timing diagram

```
module JK( input clock, J, K, reset,
      output reg Q );
   always @( posedge reset,
         posedge clock )
      casex ( { reset, J, K } )
         'b1xx: Q <= 0;
         'b000: Q \le Q;'b001: Q <= 0;
         'b010: Q \leq 1;'b011: Q \leq Q;endcase
```
 $Q(t+1)$ J K  $0<sub>0</sub>$  $Q(t)$  $0<sub>1</sub>$  $\bf{0}$  $1\,0$ 1  $\overline{Q}(t)$  $\mathbf{1}$  $\mathbf{1}$ 

(b) Truth table



endmodule

(c) Graphical symbol

```
module SynchReset( input clock, reset, D,
      output Q );
   // Synchronous reset (synchronized to the clock)
   always @( posedge clock )
      Q \le reset ? 0 : D;
```
endmodule

module AsyncReset( input clock, reset, D, output Q );

// Asynchronous reset (not synchronized to the clock) always @( posedge reset, posedge clock )  $Q \le$  reset ? 0 : D;

endmodule

D flip-flops with synchronous and asynchronous resets.

```
module CounterA(
      input clock, reset,
      input [ 31:0 ] resetValue,
      output reg [31:0] count = 0 );
   // Synchronous reset (synchronized to the clock)
   always @( posedge clock )
      count \le reset ? resetValue : count + 1;
```
endmodule

```
module CounterB(
      input clock, reset,
      input [ 31:0 ] resetValue,
      output reg [31:0] count = 0 );
```
// Asynchronous reset (not synchronized to the clock) always @( posedge reset, posedge clock ) count  $\le$  reset ? resetValue : count + 1;

endmodule

From simulation, reset in *synchronous* in CounterA, changing only *with* the clock, and *asynchronous* in CounterB.



```
always @( posedge clock )
    count <= reset ?
      resetValue : count + 1;
```
#### **CounterA CounterB**

```
always @( posedge reset, posedge clock )
    count <= reset ?
      resetValue : count + 1;
```

```
module ShiftRegister #( parameter n = 4 )
      ( input clock, reset, D,
      output reg [0: n - 1] Q );
   always @( posedge clock )
      if ( reset )
         Q \leq 0;
      else
         begin
         integer i;
         for ( i = n - 1; i = 0; i = i - 1 )
            Q[ i ] \le Q[ i - 1 ];
         Q[ 0 ] \leq D;end
```
endmodule

```
module ShiftRegister2 #( parameter n = 4 )
      ( input clock, reset, D,
      output reg [0: n-1] Q;
   always @( posedge clock )
      if ( reset )
         Q \leq 0;
      else
         Q \leq \{ D, Q[0 : n - 2 ] \};endmodule
```
### Finite state machines

Machines whose next state depends on the inputs and the previous state.

Called finite state machines because they have only a *finite* number of states.

In a combinational circuit, the values of the outputs are determined solely by the present values of its inputs.

In a sequential circuit, the values of the outputs depend on the past behavior of the circuit, as well as the present values of its inputs.

A sequential circuit has states, which in conjunction with the present values of inputs determine its behavior.

Sequential circuits can be:

• Synchronous – where flip-flops are used to implement the states, and a clock signal is used to control the operation

• Asynchronous – where no clock is used

# Terminology

*Sequential circuits:* Outputs depend on the past behavior as well as the present inputs.

*Synchronous sequential:* Controlled by a clock.

*Asynchronous sequential:* No clock is used.

**State:** The stored values of any flip-flops.

*Active edge:* The edge of the clock that causes the outputs to change.

*Finite state machine (FSM):* Formal name for a sequential circuit.

### Generalized form of a sequential circuit



If the outputs depend on both the current state and the current inputs, it is called a *Mealy* machine, named after George Mealy, who invented the concept in 1955.

### Moore machine



If the outputs depend only on the current state, it is called a *Moore* machine. *("Moore is less.")* It's named after Edward Moore, who invented the concept in 1956.



*Mealy* machines require fewer states but if the inputs change asynchronously, the outputs can change asynchronously as well.



*Moore* machines require more state variables and the outputs are delayed by one clock. But all the outputs are guaranteed to be synchronous.

Asynchronous inputs and metastability

Two related problems.

- 1.*Asynchronous inputs.* An input may change be between clocks.
- 2. *Metastability.* The input might not actually be a 1 or a 0.

If an input signal with one of these problems is a combinatorial factor in your output, your output will have the same problems.

# Solution

We can condition the input somewhat with analog circuitry, e.g., Schmidt triggers.

The digital solution is that we can sample it with a clock and save the result in a flip-flop.

We expect that no matter what the input, the flip-flop will certainly settle to 1 or 0 and that's usually true.

## But note

Sampling doesn't always work.

- It's possible for a flip-flop to get stuck in a metastable state because of a bad input, never settling into a 0 or 1.
	- If you add additional flip-flops to create a shift register, you can reduce the likelihood of having a metastable circuit but you cannot make it go away.

Simplest example: A counter



State diagram for a counter with 4 states.

At each clock, it always moves to the next state.

The arrows between states are called *edges* or *transitions* .









We pick any state assignments we like, though some could be better than others.

#### *State-assigned* **table**

**State table**







Having picked the assignments, we can use Karnaugh maps to derive the equations for the next state variables.



 $Y1 = y1'$ 

### *State-assigned* **table**



**Y2** y1 0 1 y2 0 0 1 1 1 0

 $Y2 = y1 \wedge y2$ 



 $Y2 = y1 \wedge y2$ 



 $Y2 = y1 \wedge y2$ 

module Counter( input clock, reset, output reg y1, y2 ); always @( posedge reset, posedge clock ) if ( reset ) begin  $y1 \le 0;$  $y2 \le 0;$ end else begin  $y1 \leq y1$ ;  $y2 \le y1 \wedge y2$ ; end endmodule

In Verilog, the <= "non -blocking" assignment means all the assignments happen synchronously at exit from the always block.

In Verilog, we would probably skip the Karnaugh maps write the code directly from the state-assigned

### *State-assigned* **table**



```
table. table. The counter of the module Counter 2 ( input clock,
                                     reset,
                                     output reg [ 1:0 ] y );
                                  always @( posedge reset,
                                        posedge clock )
                                     y \le reset ? 0 : y + 1;
                               endmodule
```
If the cases were more complex or not in order, we might write it like this with a case statement.

*State-assigned* **table**



```
module Counter3( input clock,
      reset,
      output reg [ 1:0 ] y );
```

```
always @( posedge reset,
   posedge clock )
      if ( reset )
         y \le 0;
      else
         case ( y )
            0: y \le 1;
            1: y \le 2;
            2: y \le 3;
            3: y \le 0;endcase
```
endmodule

Or we might parameterize the assignments.

### module Counter4( input clock, reset, output reg [ 1:0 ] y );

```
parameter A = 0, B = 1,
   C = 2, D = 3;
```

```
always @( posedge reset,
      posedge clock )
   if ( reset )
      y \le 0;
   else
      case ( y )
         A: y \leq B;
         B: y \le C;
         C: y \le D;D: y \leq A;
      endcase
```
### endmodule

### *State-assigned* **table**



Verilog makes it really easy to pick any assignments you like and walk from one arbitrary state to another.

*Example:* Create a 3-bit counter in Verilog that cycles through this sequence: 4, 7, 0, 3, 2, 6, 1, 5.

```
Example: Create a 3-bit counter in Verilog that 
cycles through this sequence: 4, 7, 0, 3, 2, 6, 1, 5.
```

```
module PseudoRandom( input clock,
    output reg [ 2:0 ] Q ;
  always @( posedge clock )
    case ( Q )
      4 \t0 \leq 77: Q \leq 0;0: Q \leq 3;3 \tQ \leq 2;2: Q \le 6;6: Q \le 1;
      1: Q \leq 5;5: Q \leq 4;endcase
```
endmodule

#### Reset



In the previous example, we didn't specify the outputs, merely that it had to count.

But consider this state diagram, where the outputs have been specified.

The "/*nnnn*" part specifies the desired outputs at each state.

Since the outputs depend only on the state, this is a Moore machine.
Reset







#### **State-assigned table**





#### **"One hot" state-assigned table**



So which is the better design? Both are correct.

Picking a good set of state assignments usually involves consideration of both cost and elegance.





# Example: A simple speed control

Outputs 1 if a vehicle's speed is excessive for 2 or more clocks.

*Example:* A speed governor that limits a vehicle's top speed.

*Input:*

- $w == 1 \rightarrow e$ xcessive speed
- $w == 0 \rightarrow$  speed acceptable

*Output:*

 $z == 1 \rightarrow$  reduce speed

 $z == 0 \rightarrow \text{maintain current speed}$ 

 $w == 1$  for 2+ clocks  $\rightarrow z == 1$ 



Figure 6.2. Sequences of input and output signals.



w == 1 for 2+ clocks  $\rightarrow$  z == 1

Figure 6.3. State diagram for the speed control.



#### Figure 6.4. State table for the speed control.



Figure 6.5. A generalized solution to the speed controller, with input *w*, output *z*, and two flip-flops for the three states.



The present state variables,  $y_1$  and  $y_2$ , determine the present state of the circuit.

The next state variables,  $Y_1$  and  $Y_2$ , determine the state into which the circuit will go after the next active edge of the clock signal.

## State variable assignments

Each of the states in a state diagram or a state table must be represented by some unique combination of 1's and 0's.

We have to pick those assignments and some assignments are better than others.



Figure 6.6. One possible state assignment for the speed controller.



Figure 6.7. Karnaugh maps for the next state variables in the speed controller.



Figure 6.8. Final implementation of the speed controller using the don't cares.



Figure 6.9. Timing diagram for the speed controller.

Design steps:

- 1. Obtain the specification of the desired circuit.
- 2. Derive a state diagram.
- 3. Derive the corresponding state table.
- 4. Reduce the number of states if possible.
- 5. Decide on the number of state variables.
- 6. Choose the type of flip-flops to be used.
- 7. Derive the logic expressions needed to implement the circuit.

### Are all state assignments equivalent?

No, they are not.



Figure 6.16. Improved state assignment for the speed controller.



#### Figure 6.17. Final circuit for the improved state assignment for the speed controller.









Original versus improved state assignment for the speed controller.



```
module simple1 ( input clock,
      reset, w, output z );
   reg [ 2:1 ] y, Y;
   parameter [ 2:1 ] A = 2'b00,
      B = 2'b01, C = 2'b10;
```

```
// Define the next state
always \mathcal{Q}(w, y)case ( y )
      A: if ( w ) Y = B;else Y = A;
      B: if ( w ) Y = C;else Y = A;
      C: if ( w ) Y = C;
         else Y = A;
      default: Y = 2'bxx;
   endcase
```
// Define the sequential block always @( posedge reset, posedge clock) if ( reset )  $y \leq A$ ; else  $v \leq Y$ :

// Define output assign  $z = y == C$ ;

endmodule



Figure 6.29. Verilog code for the speed controller.





Figure 6.32. Simulation results for the speed controller.

```
module simple2 ( input clock,
// Define the sequential block
     reset, w, output reg z );
```

```
reg [ 2:1 ] y, Y;
parameter [ 2:1 ] A = 2'b00,
   B = 2'b01, C = 2'b10;
```

```
// Define the next state
always @( w, y )
   begin
   case ( y )
      A: if ( w ) Y = B;else Y = A;
      B: if ( w ) Y = C;
         else Y = A;
      C: if ( w ) Y = C;
         else Y = A;
      default: Y = 2'bxx;endcase
   z = y == C; // Define output
   end
```

```
always @( posedge reset,
      posedge clock)
   if ( reset ) y \leq A;
   else v \leq Y:
```
#### endmodule



Figure 6.33

```
module simple3 ( input clock,
// Define output
      reset, w, output z );
   reg [ 2:1 ] y, Y;
   parameter [ 2:1 ] A = 2'b00,
      B = 2'b01, C = 2'b10;
   // Define the next state
   always @( posedge reset,
         posedge clock)
      if ( reset )
         y \leq A;
      else
         case ( y )
            A: if ( w ) y \leq B;
               else y \leq A;
            B: if ( w ) y \le C;
               else y \leq A;
            C: if ( w ) y \le C;else y \leq A;
            default: y \leq 2'bxx;
         endcase
                                         assign z = y == C;
                                      endmodule
                                           Next state and reset 
                                           calculations moved into 
                                           the always block.
                                                      C/z=1Reset 
                                       w = 0 \bigvee A/z = 0 B/z = 0w = 1w = 1 
                                                      w = 0w=0 \bigvee w=1Figure 6.34
```

```
module simple4 ( input clock,
      reset, w, output reg z );
   reg [ 2:1 ] y, Y;
   parameter [ 2:1 ] A = 2' b00,B = 2'b01, C = 2'b10;
   // Define the next state
   always @( posedge reset,
         posedge clock)
      casex ( { reset, w, y } )
         3'b1xxx: y \leq A;
         3' b00xx: y \leq A;
         \{ 2'b01, A \}: y \le B;
         {2'b01, B}: y \le C;\{ 2'b01, C \}: y \le C;
         default: y \leq 2'bxx;
      endcase
```
// Define output assign  $z = y == C$ ;

endmodule



```
module simple5 ( input clock,
      reset, w, output req z );
```

```
reg [ 2:1 ] y, Y;
parameter [ 2:1 ] A = 2' b00,B = 2'b01, C = 2'b10,
  x = 2'bxx:
```

```
// Define the next state
always @( posedge reset,
      posedge clock )
   casex ( \{ reset, w, y \} )
      \{2'b1x, x\}: y \leq A;
      {2^{\prime}b00, x}: y \leq A;
      {2'bol, A}: y \le B;\{ 2'b01, B \}: y \le C;
      \{2'b01, C\}: y \le C;
      default: y \leq x;
```
// Define output assign  $z = y == C$ ;

endmodule



endcase



Figure 6.20. Anther alternative: A one-hot state assignment for the speed controller.

### A Mealy alternative

Clockcycle: $t_0$ $t_1$ $t_2$ $t_3$ $t_4$ $t_5$ $t_6$ $t_7$ $t_8$ $t_9$ $t_{10}$						
		W: 0 1 0 1 1 0 1 1 1 0 1				
		Z: 0 0 0 0 0 1 0 0 1 1 0				

The original Moore design



The alternative Mealy design

Figure 6.22. Sequence detector for an alternate speed controller. This one recognizes  $w = 1$  on two cycles immediately.





### Figure 6.23. State diagram of the alternate speed controller.







Figure 6.24. State table for the alternate speed controller.





Figure 6.25. State-assigned table for an alternate Mealy speed controller.



(a) Circuit



Figure 6.26. The alternate Mealy speed controller.





#### The original Moore design



#### The alternative Mealy design




The alternative Mealy design

```
module mealy1( input clock, reset, w,
                                                     B: if ( w )
                                                           begin
   output reg z );
                                                           z = 1;
reg y, Y;
                                                           Y = B;
parameter A = 0, B = 1;
                                                            end
                                                        else
// Define the next state and outputs
                                                           begin
always @( * )
                                                           z = 0;
   case ( y )
                                                           Y = A;
      A: if ( w )
                                                           end
             begin
                                                     endcase
             z = 0;
             Y = B;
                                              // Define the sequential block
             end
                                               always @( posedge reset,
         else
                                                     posedge clock )
                                                  if ( reset )
             begin
             z = 0;
                                                     y \leq A;
             Y = A;
                                                  else
             end
                                                     y \leq Y;
                    Reset
                                           endmodule
                         w = 1/z = 0w = 1/z = 1w = 0/z = 0в
                                                         Figure 6.36
```
 $w = 0/z = 0$ 

```
module mealy2( input clock, reset, w,
                                              // Define the sequential block
                                                 always @( posedge reset,
   output reg z );
                                                       posedge clock )
reg y, Y;
                                                    if ( reset )
parameter A = 0, B = 1;
                                                       y \leq A;
                                                    else
// Define the next state and outputs
                                                       y \leq Y;
always @( * )
   if ( \sim w )
                                             endmodulebegin
      z = 0;
      Y = A;
      end
                                         Reset
   else
                                               w = 1/z = 0case ( y )
         A: begin
                        w = 0/z = 0w = 1/z = 1\overline{B}A
             z = 0;
             Y = B;
             end
                                               w = 0/z = 0B: begin
             z = 1;
             Y = B;
             end
      endcase
```

```
module mealy3( input clock, reset, w,
   output z );
reg y, Y;
parameter A = 0, B = 1;
// Define the next state
always @( * )
   if ( \sim w )
                                          Reset
      Y = A;
   else
                                                w = 1/z = 0case ( y )
         A: Y = B;w = 0/z = 0w = 1/z = 1A
                                                            B
         B: Y = B;endcase
                                                w = 0/z = 0assign z = y == B && w;
// Define the sequential block
always @( posedge reset,
      posedge clock )
   y \le reset ? A : Y;
```
endmodule

```
module mealy4( input clock, reset, w,
   output z );
reg y;
parameter A = 0, B = 1;
always @( posedge reset,
      posedge clock )
   y \le ( reset | \sim w ) ? A : B;
```
assign  $z = y == B$  && w;

endmodule





## Figure 6.37. Simulation results for the Mealy machine.



Figure 6.38. Potential problem with asynchronous inputs to the Mealy speed controller.